Performance Comparison of Group Chain Sampling Plan and Modified Group Chain Sampling Plan Based on Mean Product Lifetime for Rayleigh Distribution

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Abstract

The performance of a sampling plan from group sampling family is measured by its minimum number of groups and probability of lot acceptance. Basically, once the minimum number of groups is determined, the corresponding probability of acceptance can be obtained for various sets of design parameters. This article compares the performance of two acceptance sampling plans namely group chain sampling plan (GChSP) and modified group chain sampling plan (MGChSP-1) based on the mean product lifetime for Rayleigh distribution. GChSP and MGShSP were developed based on the operating procedure in both chain sampling plan (1955) and group sampling plan (2009). The findings proved that the MGChSP performed better than the GChSP.

Keywords: *Rayleigh distribution, Mean product lifetime, Chain sampling, Probability of Lot Acceptance.*

Introduction

Lifetime (or durability) and reliability are two common dimensions of product quality. Lifetime of a product refers to a statistical estimate of how long a product is predicted to carry out its predestined function, while reliability is a statistical estimate of the ability of the product to perform its intentional function for a specific time interval. Both lifetime and reliability testing are evaluated under specific set of environmental and technical conditions. In acceptance sampling, mean life is a common parameter used in evaluating the lifetime of a product. A product is defined as good when its true mean life equals or exceeds the value of its pre-specified mean life.

In most situations, a product would have a very high value of lifetime which will cause time wasting to wait until a defective item is found in the inspected sample. To solve this shortcoming, time truncated life test is introduced where the inspection process is associated with a specified interval of time. The observation of defective items in the sample will be terminated either when the number of defectives observed exceeds the acceptance number before a pre-assigned termination time, or when that termination time is reached.

The main concerns in developing various kinds of sampling plan are inspection time, inspection cost and probability of lot acceptance. The inspection time and cost are directly affected by the sample size used and how many items are inspected at one time. Meanwhile, the probability of lot acceptance or also known as operating characteristic (OC) value is influenced by how the operating procedure of the sampling plan is designed.

The probability of lot acceptance, $L(p)$ can be interpreted as a function of the deviation of pre-specified quality level of a product from its true quality level. This function is commonly known as operating characteristic (OC) function. Meanwhile, operating characteristic (OC) curve is a graph plot of probability of lot acceptance, $L(p)$ versus quality of a product. It is used as a main tool to measure the performance of a sampling plan in terms of $L(p)$, producer's risk, α and consumer's risk, β. OC curve is unique to a specific sampling plan of choice. As depicted in Figure 1, the quality of the product is described by the proportion defective. A lot is regarded as good if the proportion defective is less than the acceptable quality level (AQL), while it is considered as bad if the value of proportion defective exceeds the lot tolerance percent defective (LTPD).

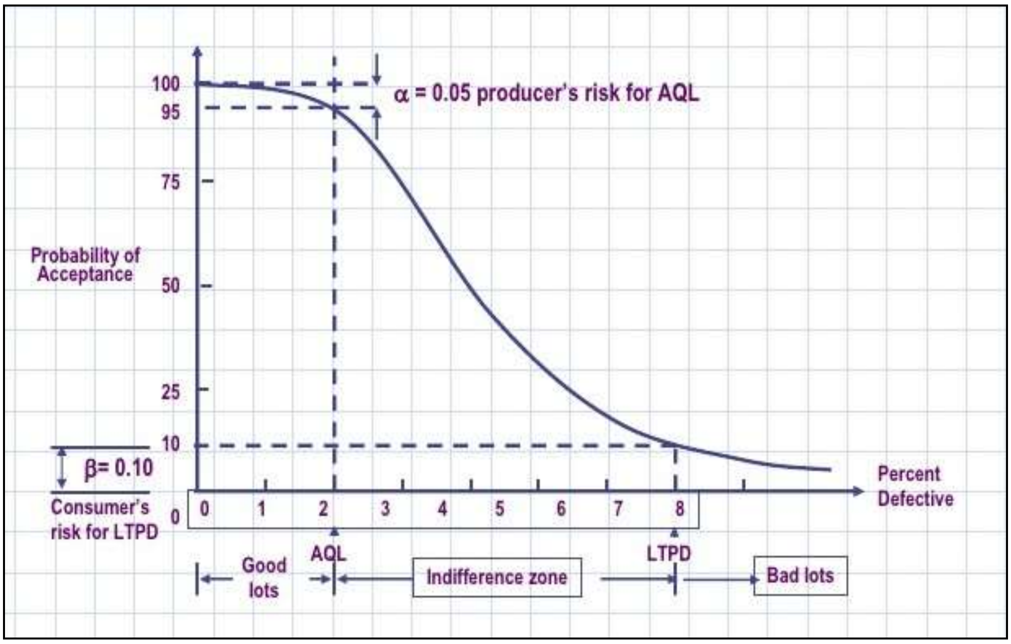

Figure 1: Example of OC curve

Source: Adapted from "operating characteristics curve", In SlideShare, Retrieved Oct 20, 2019, from http://www.slideshare.net/chintantrivedi77/operating-characteristics-curve. Copyright 2014 by RCOEM.

By definition, AQL is the lowest quality level of supplier's process that is acceptable by the consumer, which is often based on the process average. Meanwhile, LTPD represents the worst quality level of a product that is acceptable in a single lot. Since acceptance sampling involves inspecting a sample instead of the whole lot, there are chances of a bad lot being accepted (consumer's risk) and a good lot being rejected (producer's risk).

It can be observed in Figure 1 that the value of producer's risk is coupled with AQL while the value of consumer's risk is coupled with LTPD. These four values (AQL, LTPD, producer's risk & consumer's risk) become the main concerns in selecting acceptance sampling plans to be implemented by referring to the OC curve. For instance, the $L(p)$ in a sampling plan with zero acceptance number will fall sharply with just little increment of proportion defective. Consequently, this gives rise to producer's risk, which is rather unfair to the producer.

Evolution of Sampling Plans

Single sampling plan (SSP) is considered the simplest sampling plan. SSP with acceptance number equal to zero has been developed by providing a very tight inspection in order to give more protection to the consumer. Unfortunately, it has an obvious shortcoming where the probability of lot acceptance drops at a very fast rate corresponding to the small increase of proportion defective, which is rather unfair to the producer.

Many researchers have established acceptance sampling plans using attribute data by following different kinds of distribution. Formerly, [1] introduced SSP by considering the lifetime of a product which follows exponential distribution. [2] used inverse Rayleigh distribution, while [3] and [4] considered Rayleigh and log-logistic distributions, respectively for their proposed single sampling plans. Later, the development of SSP is continued by [5] and [6] by testing Marshall-Olkin extended Lomax and Marshall-Olkin extended exponential distribution as their lifetime distributions.

To overcome the shortcoming of SSP with zero acceptance number, [7] suggested chain sampling plan (ChSP-1) by making use of cumulative data of more than one sample. Its distinguished feature is that the acceptance criterion of the current lot is based on results of the immediately preceding samples. By considering zero acceptance number for both plans, it is found that ChSP-1 boosts the probability of lot acceptance of normal SSP with zero acceptance number at good quality level, but maintain the probability of lot acceptance at low quality level. In conclusion, ChSp-1 provides better protection to the producer for this case [8]. Ramaswamy and Jayasri [9-12] proposed ChSP-1 when the lifetime of the product follows exponential, Marshall-Olkin extended Exponential (MOEE), Rayleigh and inverse Rayleigh distribution respectively.

Later, [8] designed a modified chain sampling plan (MChSP-1) by offering tighter condition of sample inspection compared to the previous ChSP-1. Particularly, MChSP-1 performs better than ChSP-1 in terms of consumer's sake. It is found that MChSp-1 reduces the probability of lot acceptance of normal SSP with zero acceptance number at poor quality level while maintains the probability of lot acceptance at good quality level. In addition, MChSP-1 generates smaller sample size if compared to ChSP-1 which means MChSP-1 is better in saving the time and cost of inspection.

Next, group sampling plan (GSP) was introduced to fulfill the desire of testing more than one item at one time. This is understandable because logically by testing multiple items simultaneously, the inspection time and cost can be reduced. The performance of this type of sampling technique is measured by the minimum number of groups and the probability of lot acceptance. [13-17] respectively used Rayleigh, inverse Rayleigh, exponential, MOEE and MOEL distributions in developing GSP based on time truncated life test. Recently, [18] developed GSP for Pareto distribution of the 2nd kind.

The idea of group chain sampling plan (GChSP) is originally introduced by [19] where they integrate the operating procedures in both group sampling plan and chain sampling plan by considering Pareto distribution of the 2nd kind as lifetime distribution. Most recently, [20] and [21] then extended the development of GChSP by using Rayleigh and log-logistic distributions respectively. Later, modified group chain sampling plan (MGChSP) was introduced by [22] for generalized condition and in the following year [23] has developed the MGChSP for Pareto 2nd kind.

This article illustrates the performance comparison (in terms of probability of lot acceptance and number of groups) of two acceptance sampling plans namely group chain sampling plan (GChSP) and modified group chain sampling plan (MGChSP-1). Application on real data set is also illustrated.

Glossary of Symbols

g	:	Number of groups
r	:	Group size
n	:	Sample size
d	:	Number of defective items
c	:	Acceptance number
α	:	Producer's risk (Probability of rejecting a good lot)
β	:	Consumer's risk (Probability of accepting a bad lot)
$L(p)$	:	Probability of lot acceptance
p	:	Proportion defective
δ	:	Scale parameter
$\frac{\mu}{\mu_0}$	:	Mean ratio
a	:	Time termination multiplier
i	:	Number of preceding lots

Rayleigh Distribution

The cumulative distribution function of Rayleigh distribution is given by

$$F(t;\delta) = 1 - exp\left[-\frac{1}{2}\left(\frac{t}{\delta}\right)^2\right]; \qquad (1)$$

$t > 0$, $\delta > 0$, where δ is the scale parameter.

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When the lifetime follows the Rayleigh distribution, the true mean life is given by $\mu = \sqrt{\frac{\pi}{2}}\,\delta$. Then, the proportion defective is given by $p = \mathrm{F}(a\mu_0\,;\,\delta)$;

$$p = 1 - exp\left[\frac{-1}{2}\left(\frac{a\mu_0}{\frac{\mu}{\sqrt{\pi/2}}}\right)^2\right] = 1 - exp\left[\left(\frac{-\pi a^2}{4}\right)\left(\frac{1}{\frac{\mu}{\mu_0}}\right)^2\right]. \qquad (2)$$

Probability of Lot Acceptance

The probability of lot acceptance for the GChSP is given as

$$L_{GChSP}(p) = P_0 + P_1(P_0)^i. \qquad (3)$$

As this is a case of independent trials which involves success and failure, Binomial expression is considered to complete the equation. Probability of success refers to the chance of having defective items in a trial, p while probability of failure refers to the chance of having non-defective items in a trial, $(1-p)$.

For example, P_0 is equal to $[p^0 \times (1-p)^n]$ whereas P_1 is similar to $[p^1 \times (1-p)^{n-1}]$.When the concept of group sampling plan is applied, then n is expressed as $g \times r$. Therefore the probability of lot acceptance for the GChSP is given by

$$L_{GChSP}(p) = (1-p)^{gr} + (1-p)^{gr-1}(1-p)^{gri}. \qquad (4)$$

The probability of lot acceptance for the MGChSP can be written as

$$L_{MGChSP}(p) = (P_0)^{i+1} + P_1(P_0)^i, \qquad (5)$$

which then upon simplification becomes

$$L_{MGChSP}(p) = (1-p)^{(gr)(i+1)}[1 + i(gr)(p)/(1-p)]. \qquad (6)$$

Findings

Comparative analysis between MGChSP and GChSP is conducted by considering minimum number of groups and probability of lot acceptance as performance indicators. The following Table 1 shows the comparison between these two types of sampling plan following Rayleigh distributions in terms of minimum number of groups.

Table 1: Minimum number of groups for MGChSP and GChSP when $i = 1$, $r = 2$ and $\beta = 0.01$

Sampling plan	a					
	0.7	0.8	1	1.2	1.5	2
GChSP	7	5	3	3	2	1
MGChSP	5	4	3	2	1	1

As depicted in Table 1, MGChSP produces smaller minimum number of groups compared to GChSP when the lifetime follows Rayleigh distribution. The following Figure 2 illustrates the comparison between MGChSP and GChSP in terms of $L(p)$ by using OC curve.

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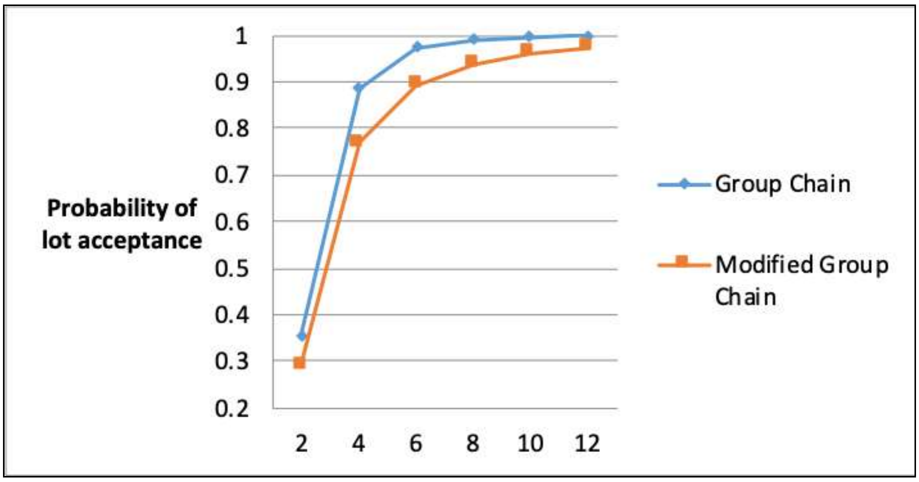

Figure 2: OC curve of MGChSP and GChSP based on Rayleigh distribution when

$a = 0.7,\ r = 2,\ i = 1,\ \beta = 0.01$

As observed in Figures 2, probability of lot acceptance of MGChSP is smaller compared to GChSP based on Rayleigh distribution. The findings in thus far have shown that MGChSP has two main advantages over GChSP based on Rayleigh distribution. First, MGChSP produces smaller sample size compared to GChSP. In group sampling technique, the equation $n = gr$ is applied which indicates the proportional relationship between minimum number of groups, g and sample size, n. In addition, sample size used is associated with the inspection time and cost. Therefore, the smaller minimum number of groups recorded by MGChSP reduces the time and cost of inspection.
Second, MGChSP provides better protection to consumer compared to GChSP. Better protection to consumer requires lower consumer's risk by having smaller $L(p)$ at low quality level. In addition, it is important to be noted that the existence of MGChSP is only when the number of groups is bigger than 1. Otherwise, the sampling plan will reduce to the normal MChSP-1 as developed by [8].

Real Data Application

Apart from earlier simulation, a real data set is also used to demonstrate the applicability of MGChSP. An appropriate lifetime distribution fit by the data set must be identified to determine the most suitable sampling plan to be applied. The data set from [24] based on number of cycles of 100 units of yarn before its failure in time truncated test is presented in Table 2.

Table 2: Number of cycles upon failure for each of 100 units of yarn

Yarn	Number of cycles upon failure	Yarn	Number of cycles upon failure	Yarn	Number of cycles upon failure	Yarn	Number of cycles upon failure
1	86	26	198	51	211	76	55
2	146	27	38	52	180	77	61
3	251	28	20	53	93	78	244
4	653	29	61	54	315	79	20
5	98	30	121	55	353	80	284
6	249	31	282	56	571	81	393
7	400	32	224	57	124	82	396
8	292	33	149	58	279	83	203
9	131	34	180	59	81	84	829
10	169	35	325	60	186	85	239

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11	175	36	250	61	497	86	236
12	176	37	196	62	182	87	286
13	76	38	90	63	423	88	194
14	264	39	229	64	185	89	277
15	15	40	166	65	229	90	143
16	364	41	38	66	400	91	198
17	195	42	337	67	338	92	264
18	262	43	65	68	290	93	105
19	88	44	151	69	398	94	203
20	264	45	341	70	71	95	124
21	157	46	40	71	246	96	137
22	220	47	40	72	185	97	135
23	42	48	135	73	188	98	350
24	321	49	597	74	568	99	193
25	180	50	246	75	55	100	188

The most appropriate distribution for the data in Table 2 is identified by the Kolmogorov- Smirnov (K-S) goodness of fit test using Easy-Fit 5.6 Software. The summary of K-S Statistic value for each lifetime distribution tested is shown in the following Table 3.

Table 3: Kolmogorov- Smirnov (K-S) goodness of fit summary

Lifetime Distribution	K-S Statistic	Lifetime Distribution	K-S Statistic	Lifetime Distribution	K-S Statistic
Beta	0.11100	Inv. Gaussian	0.10800	Normal	0.10569
Burr	0.08026	Johnson SB	0.08587	Pareto	0.35278
Cauchy	0.11979	Kumaraswamy	0.13527	Pareto 2	0.20448
Erlang	0.19402	Laplace	0.13470	Pert	0.07339
Error	0.13470	Levy	0.34714	Rayleigh	0.07122
Exponential	0.20021	Log-Gamma	0.16084	Reciprocal	0.30035
Fatigue Life	0.17733	Logistic	0.10923	Rice	0.10496
Frechet	0.21289	Log-Logistic	0.14959	Triangular	0.22521
Gamma	0.08895	Lognormal	0.14097	Uniform	0.15023
Hypersecant	0.11516	Log-Pearson 3	0.07900	Weibull	0.08425

By referring to Kolmogorov- Smirnov (K-S) Table at 1% significance level, the critical value for Kolmogorov- Smirnov (K-S) goodness of fit test is 0.3295. If K-S Statistic value a lifetime distribution is less than 0.3295, then the data in Table 2 fit that particular distribution.

Among the distributions fit by the data, the one with the lowest K-S Statistic is considered as the best. K-S Statistic for Rayleigh distribution is 0.07122 which is lower than 0.3295 and the lowest among all distributions in Table 3. Thus, Rayleigh distribution is the best fit for the product. Then modified group chain sampling plan for truncated life test based on Rayleigh distribution is applied.

The consumer's confidence level is related to consumer's risk, β. If we denote ρ as consumer's confidence level, then $\beta = 1 - \rho$. Given that Rayleigh distribution is the lifetime distribution for the product with 0.99 consumer's confidence level and at least 500 hours of true mean life are required by the experimenter. Since group sampling approach is applied, there is an advantage for the experimenter to inspect more than one item on a tester at the same time.

Assume that the experimenter takes the values of $a = 0.7$, $r = 2$ and $i = 1$ for the experiment. Then, the corresponding minimum number of groups is $g = 5$ as shown in Table 1. Since $n = gr$, the experimenter must randomly draw a sample of size 10 to test where these 10 items are distributed into 5 groups with 2 items in each group. Four possibilities will occur in the experiment:

1. During 500 hours of inspection, if no item is found defective, on condition that one preceding sample also has no defective item, then the experimenter will accept the current lot.
2. During 500 hours of inspection, if no item is found defective, on condition that one preceding sample has one defective item, then the experimenter will accept the current lot.
3. During 500 hours of inspection, if no defective item is found on condition that one preceding sample has more than one defective item, then the experimenter will reject the current lot.
4. During 500 hours of inspection, if at least one defective item is found, then the experimenter will reject the current under inspection lot without considering any preceding sample.

GChSP is also applied based on the real data set in Table 2 for comparison purposes. The performances of both GChSP and MGChSP based on the data set are measured in terms of g and $L(p)$ provided the lifetime of the yarn follows Rayleigh distribution. The generated minimum number of groups when $r = 2$ and $i = 1$ is shown in Table 4, while the $L(p)$ values when $r = 2$, $i = 1$ and $a = 0.7$ is presented in Table 5.

Table 4: Minimum number of groups for MGChSP and GChSP when lifetime of the yarn follows Rayleigh distribution

β	Sampling Plan	a					
		0.7	**0.8**	**1.0**	**1.2**	**1.5**	**2.0**
0.25	MGChSP	2	2	1	1	1	1
	GChSP	3	2	2	1	1	1
0.10	MGChSP	3	2	2	1	1	1
	GChSP	4	3	2	2	1	1
0.05	MGChSP	3	3	2	2	1	1
	GChSP	5	4	3	2	1	1
0.01	**MGChSP**	**5**	**4**	**3**	**2**	**1**	**1**
	GChSP	7	5	3	3	2	1

Table 5: Probability of lot acceptance for MGChSP and GChSP when lifetime of the yarn follows Rayleigh distribution

β	Sampling plan	g	$\frac{\mu}{\mu_0}$					
			2	4	6	8	10	12
0.25	MGChSP	2	0.6503	0.9053	0.9575	0.9760	0.9846	0.9893
	GChSP	3	0.7523	0.975	0.9946	0.9982	0.9993	0.9996
0.10	MGChSP	3	0.5062	0.8587	0.9363	0.9641	0.9770	0.9840
	GChSP	4	0.6363	0.9575	0.9905	0.9969	0.9987	0.9994
0.05	MGChSP	3	0.5062	0.8587	0.9363	0.9641	0.9770	0.9840
	GChSP	5	0.5293	0.9366	0.9854	0.9951	0.9979	0.999
0.01	MGChSP	5	0.2934	0.7686	0.8943	0.9402	0.9616	0.9733
	GChSP	7	0.3554	0.8878	0.9725	0.9906	0.996	0.998

By observing Table 4 and Table 5, it can clearly be noticed that MGChSP produces smaller g and smaller $L(p)$ compared to GChSP at four levels of consumer's risk. These findings are consistent with the comparison result as discussed in Section 6.

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Conclusions

The performance of the MGChSP is measured by the minimum number of groups, g and probability of lot acceptance, $L(p)$. The influence of several combinations of design parameters on these indicators has been investigated. Larger values of pre-specified consumer's risk, β, preceding lots, i and time termination multiplier, a contribute to a reduction in minimum number of groups, g which also means a reduction in sample size, n. Meanwhile, the $L(p)$ increases with the mean ratio. These findings are consistent with those of the established group sampling plans as studied by [13-17].
For Rayleigh distribution, the comparison between MGChSP and group chain sampling plans (GChSP) has shown that MGChSP produces lower minimum number of groups, g compared to GChSP. Consequently, the required sample size $(n = gr)$ is reduced, and so are the inspection time and cost. As portrayed in Section 6 and 7, MGChSP generates lower probability of lot acceptance compared to GChSP, which provides better protection to the consumer by reducing the risk of accepting bad lots, β. Both performance indicators prove that the proposed MGChSP consistently performs better than the established GChSP.

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